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Industrial Training and

Analytical model of LPP

$$z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

STC $a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n = b_1$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$

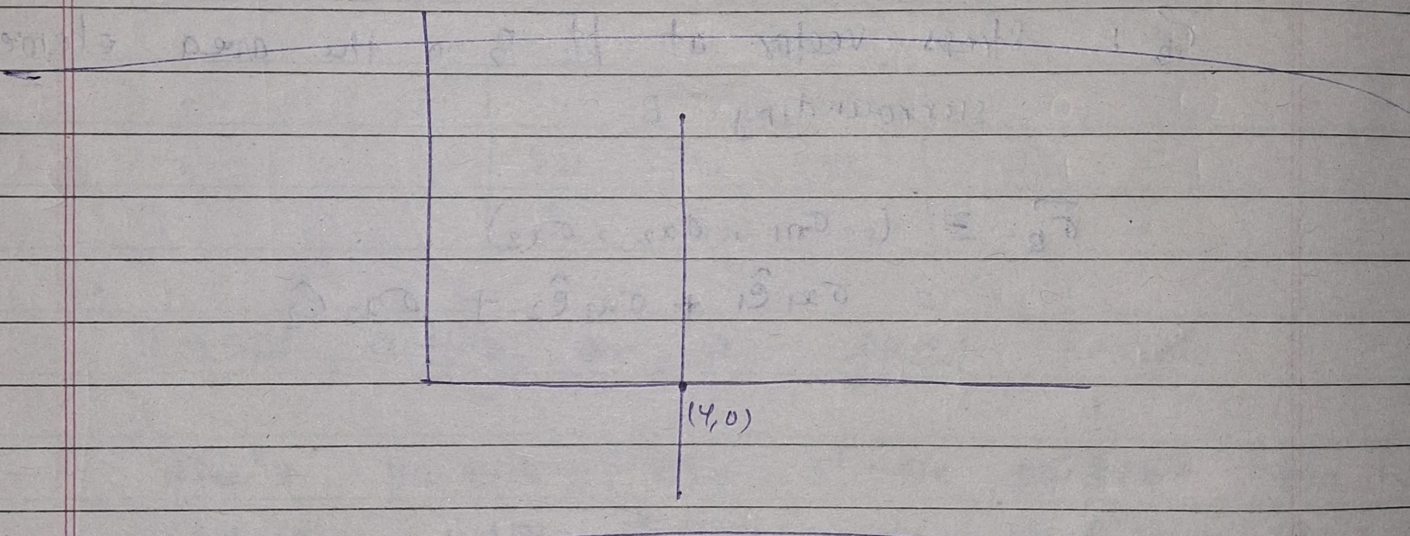
Max $Z = 3x_1 + 5x_2$ — (0)

ST. $x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 \le 18$ — (3)

$x_i \geq 0, \ i = 1 \text{ to } 2$



$Z = 3x_1 + 5x_2 + 0x_3 + 0x_4 + 0x_5$

ST! $x_1 + \quad + x_3 = 4$ — (1a)

$2x_2 \quad + x_4 = 12$ — (2a)

$3x_1 + 2x_2 \quad + x_5 = 18$ — (3a)

$x_i \geq 0$ for $i = 1$ to 5

Choosing the basic soln as (0,0)

Basic feasible solution (0,0,0,0,0)

Retain $x_1 = 0$

1(a) — $x_3 = 4$

1(b) — $2x_2 + x_4 = 12$

$x_4 = 12 - 2x_2 \Rightarrow x_2 \le 6$

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1(c) $2x_2 + x_5 - 18$

$\Rightarrow x_2 = 9$

$x_1 + x_3 = 4$ — (1b)

$x_2 + x_4/2 = 6$ — (2b)

$3x_1 - x_4 + x_5 = 6$ —

of

$Z = 30 + 3x_1 - \frac{5}{4}x_4$

(2b) $x_3 = 4 - x_1 \Rightarrow x_1 \leq 4$

(1c) —

1(d) $3x_1 + x_5 = 6$

$x_5 = 6 - 3x_1 \Rightarrow x_1 \leq 2$

$Z = 36 - \frac{3}{2}x_4 - x_5$

ST $x_3 + \frac{1}{3}x_4 - \frac{1}{3}x_5 = 2$

$x_2 + \frac{1}{2}x_4 = 6$

$x_1 - \frac{1}{3}x_4 + \frac{1}{3}x_5 = 2$

$Z_{max} = 3x_1 + 5x_2$

ST $\quad x_1 + x_3 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 < 18$

$x_i \ge 0$

linear form

$Z - 3x_1 - 5x_2 - 0s_1 - 0s_2 - 0s_3 = 0$

$x_3 + x_1 + s_1 + 0s_2 + 0s_3 = 4$

$2x_2 + 0s_1 + s_2 + 0s_3 = 12$

$3x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 18$

You first max

B.V	Z	x_1	x_2	x_3	x_4	x_5	sol^n	ratio
Z_{max}	1	-3	-5	0	0	0	0	-
x_3	0	1	0	1	0	0	4	-
x_4	0	0	2	0	1	0	12	6
x_5	0	3	2	0	0	1	18	9

max size

leaving

BV	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
Z	-3	0	0	5/2	0	30	—
x_3	1	0	1	0	0	4	4
x_2	0	1	0	1/2	0	6	—
x_5	3	0	0	-1	1	6	2

leav

BV	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
Z	0	0	0	3/2	1	36	
x_3	0	0	1	1/3	-1/3	2	
x_2	0	1	0	1/2	0	6	
x_1	1	0	0	-1/3	1/3	2	

$Z = 36$

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maximize $Z = 3x_1 + 5x_2$

ST

$x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 = 18$

$x_1 \geq 0, \ x_2 \geq 0$

~~som~~

$x_1 + x_3 = 4$

$2x_2 + x_4 = 12$

$3x_1 + 2x_2 + \bar{x}_5 = 18$

max: $Z = 3x_1 + 5x_2 - \underbrace{M\bar{x}_5}_{M \to \infty}$

$x_i \geq 0$ for $i = 1$ to 5

$x_5 = 18 - 3x_1 - 2x_2$

simplest form

$Z - \cancel{3} (3M+3)x_1 - (2M+5)x_2 = -18M$

	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
Z	3M-3	-2M-5	0	0	0	-18M	—
x_3	1	0	1	0	0	4	4
x_4	0	2	0	1	0	12	—
x_5	3	2	0	0	1	18	6

	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	0	-2M-5	3M+3				
x_1	1	0	1	0	0	4	—
x_4	0	2	0	1	0	12	6
x_5	0	2	-3	0	1	6	3

$Z = (3M+3)x_1 + (2M+5)x_2 - 18M x_5$

	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
Z	0	1	$-9/2$	0	$M+5/2$	27	36
x_1	1	0	1	0	0	4	2
x_4	0	0	3	1	-1	6	2
x_2	0	1	$-3/2$	0	$1/2$	3	6

optimal soln $x_1 = 2,\quad x_4 = 2,\quad x_2 = 6$

$x_3 = 0,\quad x_5 = 0$

$Z_{max} = 36$

Ques $0.6x_1 + 0.4x_2 \geq 6$

$0.6x_1 + 0.4x_2 - x_3 = 6$

Ques $\text{Max } Z = c_1x_1 + c_2x_2 + \cdots c_nx_n$ shadow price

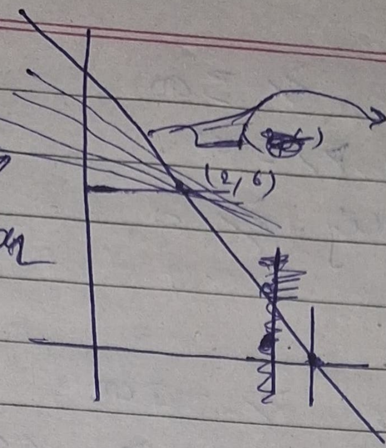
ST. $a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n \leq b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n \leq b_n$

	1	2	Avalm hrs
Plant 1	1	0	
2	0	2	
3	3	2	



$z : 45 = 7.5x_1 + 5x_2$

or

$18 = 3x_1 + 2x_2$

$z : 30 = 3x_1 + 5x_2$

$y = mx + c$

Primal

$\max z = 3x_1 + 5x_2$

S.T: $x_1 \le 4$ y_1, $2x_2 \le 12$ y_2

$3x_1 + 2x_2 \le 18$ y_3

x_1 & $x_2 \ge 0$

$2y_2 + 2y_3 = 5$

$2y_2 + y_3 \ge 5$

dual

$\text{Min } z_y = 4y_1 + 12y_2 + 18y_3$

S.T. $y_1 + 3y_3 \ge 3$

$2y_2 + 2y_3 \ge 5$

$y_1, y_2, y_3 \ge 0$

$y_1 \to x_1$, $y_2 \to x_1$ & $y_3 \to x_1$

$x_i \ge 0$

$y_i \ge 0$

↓

all unequal

↓

symmetric

No. of constraints = No. of variables, in dual

Q. a manufacturer is faced a problem of transported as ~~books~~ goods from two ware houses of min cost. they supplies at the ware houses are 300 & 600 unit to 3 ware

Soln:-

	R_1	R_2	R_3	supply
W_1	2	4	3	300
W_2	5	3	4	600
	↓	↓	↓	
demand →	200	300	500	

x_{ij} — wherehouse, retail

$\text{Min } z = 2x_{11} + 4x_{12} + 3x_{13} + 5x_{21} + 3x_{22} + 4x_{23}$

S.T.C $x_{11} + x_{12} + x_{13} \le 300$

$x_{21} + x_{22} + x_{23} \le 600$

$x_{11} + x_{21} \ge 200$

$$x_{12} + x_{22} \geq 300$$
$$x_{13} + x_{23} \geq 500$$
$$x_{ij} \geq 0$$

Now

y_1 $\quad -x_{11} - x_{12} - x_{13} \geq -300$

y_2 $\quad -x_{21} - x_{22} - x_{23} \geq -600$

y_3 $\quad x_{11} + x_{21} \geq 200$

y_4 $\quad x_{12} + x_{22} \geq 300$

y_5 $\quad x_{13} + x_{23} \geq 500$

$$x_{ij} \geq 0$$

Dual

Max $\quad y = -300y_1 - 600y_2 + 200y_3 + 300y_4 + 500y_5$

S.T.

$$-y_1 + y_3 \leq 2$$
$$-y_1 + y_4 \leq 4$$
~~# Max z = $c_1x_1 + c_2x_2 + \dots + c_nx_n$~~ $\quad -y_2 + y_3 \leq 5$

~~ST: $a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n \leq$~~ $\quad -y_1 + y_5 \leq 3$

$$-y_2 + y_4 \leq 3$$
$$-y_2 + y_5 \leq 4$$
$$y_i \geq 0$$

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$$\text{Min } z = -2x_1 - 3x_2 + 4x_3$$

ST $\quad 3x_1 + 6x_2 - 7x_3 \geq 2 \quad$ — ①

$2x_1 + 5x_2 + 4x_3 = 4$

$x_1, x_2, x_3 \geq 0$

$2x_1 + 5x_2 + 4x_3 \geq 4$

$2x_1 + 5x_2 + 4x_3 \leq 4$

$2x_1 + 5x_2 + 4x_3 \geq 4 \quad$ — ②

$-2x_1 - 5x_2 - 4x_3 \geq -4 \quad$ — ③

$x_1, x_2, x_3 \geq 0$

Now $\quad \min Z = -2x_1 - 3x_2 + 4x_3$

ST $\quad 3x_1 + 6x_2 - 7x_3 \geq 2 \qquad y_1$

$2x_1 + 5x_2 + 4x_3 \geq 4 \qquad y_2$

$-2x_1 - 5x_2 - 4x_3 \geq -4 \qquad y_3$

Dual

$\min Y = 2y_1 + 4y_2 - 4y_3$

ST: $\quad 3y_1 + 2y_2 - 2y_3 \leq -2$

$6y_1 + 5y_2 - 5y_3 \leq -3$

$-7y_1 + 4y_2 - 4y_3 \leq 4$

$y_1, y_2, y_3 \geq 0$

$\max w = 2y_1 + 4y_3 - 4y_2$

ST $\quad 3y_1 - 2y_2 + 2y_3 \leq -2$

$5y_1 - 5y_2 + 5y_3 \leq -3$

$-7y_1 - 4y_2 + 4y_3 \leq 4$

$-2(y_2 - y_3)$

$-5(y_2 - y_3)$

$-4(y_2 - y_3)$

Dual $\quad$ let $(y_2 - y_3) = y^*$

$\max.\ W = 2y_1 + 4y^*$

ST $\quad 3y_1 - 2y^* \leq -2$

$6y_1 - 5y^* \leq -3$

$-7y_1 - 4y^* \leq 4$

$y_1 \geq 0$

$y^* \rightarrow$ unrestricted.

$$\text{Min } z = -2x_1 - 3x_2$$

S.T. $3x_1 + 6x_2 \geq 2$

$2x_1 + 5x_2 \geq 4$

$x_1 \geq 0$, x_2 unrestricted

Sol^n:- ~~Min~~ let $x_2 = x_3 - x_4$

$$Z_{min} = -2x_1 - 3x_3 + 3x_4$$

ST $3x_1 + 6x_3 - 6x_4 \geq 2$ $\quad y_1$

$2x_1 + 5x_3 - 5x_4 \geq 4$ $\quad y_2$

x_1, x_3 & $x_4 \geq 0$

if primal has an equality const, dual has an unrestricted variable & vice versa

Dual

$$\text{Max } Z = 2y_1 + 4y_2$$

ST $3y_1 + 2y_2 \leq -2$

$6y_1 + 5y_2 \leq -3$

$-6y_1 - 5y_2 \leq 3$

$6y_1 + 5y_2 = -3$

If the value of a feasible solution of $f(x)$ in the minimization form

$$\underset{\text{Min}}{f(x)} \geq \underset{\text{max.}}{W(y)}$$

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	x_1	x_2	x_3	..	x_n	RHS
y_1	a_{11}	a_{12}	a_{13}	...	a_{1n}	$\le b_1$
y_2	a_{21}	a_{22}	a_{23}	..	a_{2n}	$\le b_2$
⋮	⋮				⋮	⋮
y_n	a_{m1}	a_{m2}	a_{m3}	..	a_{m}	$\le b_n$
R.H.S	$\ge$	$\ge$	...	...	$\ge$	
	c_1	c_2			c_n	

$$W = \sum_{i=1}^{m} b_i y_i$$

$$Z_i = \sum_{i=1}^{m} a_{iy} \cdot y_i \qquad y = 1 \text{ to } n$$

Ques

	x_1	x_2	
y_1	1	0	≤ 4
y_2	0	2	≤ 12
y_3	3	2	≤ 18
	$\ge$	$\ge$	
	3	5	

Row 0	x_1	x_2	x_3	...	x_n	x_{n+1} ---	x_{n+m}	RHS
← Z	Z_1-C_1	Z_2-C_1	Z_3-C_1	..	Z_n-C_1	y_1	y_m	

any iteration

$n \to$ no of decision variables

$m \to$ No. of slack/samplus/ artifical variable

$$Z_i = \sum_{i=1}^{m} a_{iy} y_i$$

primal

$$\text{Max } Z = 3x_1 + 5x_2$$

ST.

$$x_1 \le 4$$
$$2x_2 \le 12$$
$$3x_1 + 2x_2 \le 18$$
$$x_1 \,\&\, x_2 \ge 0$$

Dual

$$\text{Min } W = 4y_1 + 12y_2 + 18y_3$$

$y_1 + 3y_3 \geq 3 \longrightarrow x_1$

$2y_2 + 2y_3 \geq 5$

$y_1, y_2 + y_3 \geq 0$

for decision variable x.

$y_1 + 3y_3 \geq 3$

$y_1 + 3y_3 - S_1 = 3$

$S_1 = \underbrace{(y_1 + 3y_3)} - 3$

$= (Z_1 - C_1)$

Ques 1 $\quad \text{Max } Z = 3x_1 + 5x_2$

S.T.

$x_1 \leq 4 \longrightarrow x_1 + x_3 = 4$

$2x_2 \leq 12 \qquad 2x_2 + x_4 = 12$

$3x_1 + 2x_2 \leq 18 \quad - \quad 3x_1 + 2x_2 + x_5 = 18$

$x_1, x_2 \geq 0$

Soln: dual

$\text{min } W = 4y_1 + 12y_2 + 18y_3$

ST $\quad y_1 + 3y_3 \geq 3$

$y_2 + 2y_3 \geq 5$

$y_1, y_2, y_3 \geq 0$

Iti	Primal					Z	Dual			Slack		W
	x_1	x_2	x_3	x_4	x_5		y_1	y_2	y_3	$(Z_1 - C_1,$	$Z_2 - C_2)$	
0	-3	-5	0	0	0	0	0	0	0	-3	-5	
1	-3	0	0	5/2	0	30	0	5/2	0	-3	0	
2	0	0	0	3/2	1	36	0	3/2	1	0	0	

$$Z(x^*) = W(y^*)$$

Primal $\qquad\qquad$ Dual

$Z(x)$ $\qquad\qquad$ $f(y)$

$Z(x) = f(y)$

BFS $\qquad$ may or may not be feasible

if $f(y) \rightarrow y^* \rightarrow$ shadow

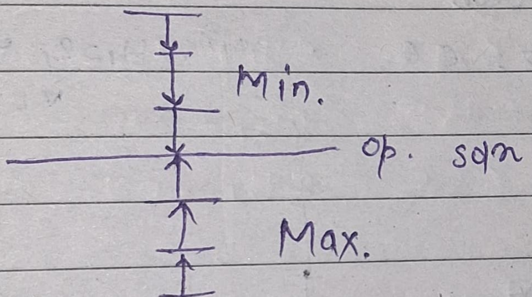
$\quad$ feasible

$Z(x^*) = f(y^*) \longrightarrow$ feasible

$f(y) \geq Z(x)$

$\underbrace{\text{Min} \quad \text{Max}}$

x & y are BFS



- If the problems is unbounded, dual is infeasible
- If primal is infeasible, dual is unbounded or has no feasible solution

Primal

Min $Z(x) = 2x_1 + 11x_2$

ST. $-x_1 \geq -5$

$2x_1 + 3x_2 \geq 6$

$x_1 \& x_2 \geq 0$

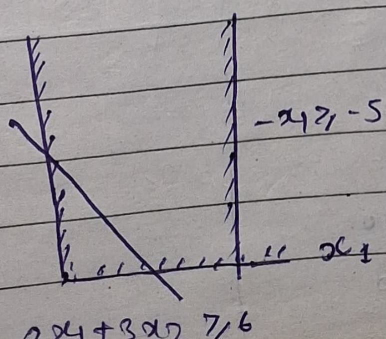
Dual

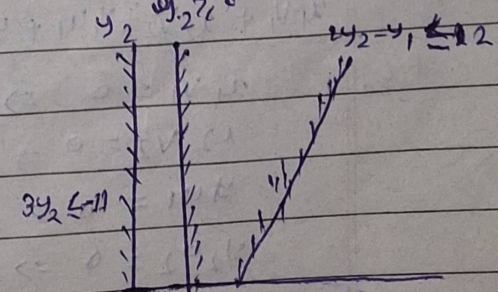
Max $W(y) = -5y_1 + 6y_2$

ST. $-y_1 + 2y_2 \leq 2$

$3y_2 < -11$

$y_1 \& y_2 \geq 0$





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Complementary Slackness Theorem

- if x^* and y^* are optimal solutions of primal & dual & u^* & v^* are their corresponding slack variables then

$$x^* \times V^* + y^* \times u^* = 0$$

- Primal
Max $10x_1 + 9x_2$
S.T $3x_1 + 3x_2 + u_1 = 21$
$4x_1 + 3x_2 + u_2 = 24$
$x_1 \,\&\, x_2,\ u_1 \,\&\, u_2 \geq 0$

Dual
min $w(y) = 21y_1 + 24y_2$
S.T. $3y_1 + 4y_2 + V_1 = 10$
$3y_1 + 3y_2 - V_2 = 9$
$y_1, y_2, V_1, V_2 \geq 0$

Soln.
optimal solution
$x_1 = 3,\ x_2 = 4,\ z = 66$
$u_1 = u_2 = 0$

soln. $y_1 = 2,\ y_2 = 1,\ w = 66$
$V_1 = V_2 = 0$

~~dual~~
~~min $21y_1 + 24y_2 = w$~~

Primal
max $Z(x) = 4x_1 + 3x_2$
ST $x_1 + 2x_2 + u_1 = 7$
$3x_1 + x_2 + u_2 = 11$
$4x_1 + 5x_2 + u_3 = 26$
$x_1, x_2, u_1, u_2, u_3 \geq 0$

optical $x_1 = 3,\ x_2 = 2$
$u_1 = u_2 = 0,\ u_3 = 4$

Dual
min $w(y) = 7y_1 + 11y_2 + 26y_3$
ST $y_1 + 3y_2 + 4y_3 - V_1 = 4$
$2y_1 + y_2 + 5y_3 - V_2 = 3$
$y_1, y_2, y_3, V_1, V_2 \geq 0$

$$x_1 V_1 + x_2 V_2 + y_1 u_1 + y_2 u_2 + y_3 u_3 = 0$$

$x_1 V_1 = 0 \Rightarrow V_1 = 0$
$x_2 V_2 = 0 \Rightarrow V_2 = 0$
$y_1 u_1 = 0 \Rightarrow y_1 \geq 0$
$y_2 u_2 = 0 \Rightarrow y_2 \geq 0$
$y_3 u_3 = 0 \Rightarrow y_3 = 0$

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$f(x) \geq W(y)$

primal — dual

Min — Max.

x & y are feasible

Primal

Max $f(x) = 4x_1 + 3x_2$

S.T.

$$x_1 + 2x_2 + u_1 = 7$$
$$3x_1 + x_2 + u_2 = 11$$
$$4x_1 + 5x_2 + u_3 = 26$$
$$x_1, x_2, u_1, u_2, u_3 \geq 0$$

feasible soln:

$$x_1 = 11/3, \quad x_2 = 0$$
$$u_1 = 10/3, \quad u_2 = 0, \quad u_3 = \frac{34}{3}$$
$$f(x) = 44/3$$

Dual

min $W(y) = 7y_1 + 11y_2 + 26y_3$

S.T.

$$y_1 + 3y_2 + 4y_3 - V_1 = 4$$
$$2y_1 + y_2 + 5y_3 - V_2 = 3$$
$$y_1, y_2, y_3, V_1, V_2 \geq 0$$

$$y_1u_1 + y_2u_2 + y_3u_3 + x_1V_1 + x_2V_2 = 0$$

~~$y_1 u_1 = 0$~~ $\Rightarrow y_1 = 0, \quad \underset{\text{basic}}{y_2 \geq 0}, \quad y_3 = 0, \quad V_1 = 0, \quad V_2 \geq 0$

when $y_1 = 0, \; y_3 = 0, \; V_1 = 0 \Rightarrow y_2 = 4/3$ — from eqn (I)

$y_1 = 0, \; y_3 = 0 \Rightarrow -V_2 = 3 - y_2$ — (II)

$$-V_2 = 3 - 4/3 = 5/3$$
$$V_2 = -5/3$$

Max $Z = 20x_1 + 6x_2 + Px_3$

ST. $8x_1 + 2x_2 + 3x_3 \le 250$

$4x_1 + 3x_2 \le 150$

$2x_1 + x_3 \le 50$

$x_1, x_2, x_3 \ge 0$

primal soln

$x_1^* = 0, \quad x_2^* = 50, \quad x_3^* \ 50$

dual

$y_1^* = 0, \quad y_2^* = 2, \quad y_3^* = 8$

soln.

	Dual
$Z = 20x_1 + 6x_2 + Px_3$	$w = 250y_1 + 150y_2 + 50y_3$
ST $8x_1 + 2x_2 + 3x_3 \le 250$	$8y_1 + 4y_2 + 2y_3 - V_1 = 20$
$4x_1 + 3x_2 \le 150$	$2y_1 + 3y_2 + 0y_3 - V_2 = 6$
$2x_1 + x_3 \le 50$	$3y_1 + 0y_2 + y_3 - V_3 = P$

$y_1u_1 + y_2u_2 + y_3u_3 + x_1V_1 + x_2V_2 + x_3V_3 = 0$

$V_1 \ge 0, \quad V_2 = 0, \quad V_3 = 0, \quad u_1 \ge 0, \quad u_1 = 0, \quad u_3 = 0$

$3 \times y_1 + 0y_2 + y_3 - V_3 = P$

$8 - 0 = P$

$\boxed{P = 8}$

	Product time/batch		
Plant	1	2	availble time
1	1	0	4
2	0	2	12
3	3	2	18
Profit/batch	3	5	

Primal

$$\text{Max } Z(x) = 3x_1 + 5x_2$$

S.T. $x_1 \le 4$

$2x_2 \le 12$, $x_1 \,\&\, x_2 \ge 0$

$3x_1 + 2x_2 \le 18$

$$\text{Max } c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

S.T. $\sum_{i=1}^{n} a_{ij}x_j = b_i$

b_i = availble resource

x_j = level of activity (production)

~~a_{ij} = profit/ unit~~

a_{ij} - units of resources i being used for activity j

~~y_i cost of~~

c_j = profit/ unit of activity j

Z = value of optimization function

$y_i \rightarrow$ cost of a unit

- Dual

$$\min W(y) = 4y_1 + 12y_2 + 18y_3$$

S.T. $y_1 + 3y_3 \ge 3$

$y_2 + 2y_3 \ge 5$

$y_1, y_2 \,\&\, y_3 \ge 0$

$$\text{Min } W(y) = b_1y_1 + b_2y_2 + \cdots + b_my_m$$

S.T. $\sum_{i=1}^{m} a_{ij}y_i = c_j$